PERGAMON

International Journal of Solids and Structures 36 (1999) 4609–4627

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Transient analysis of a propagating crack with finite length subjected to a horizontally polarized shear wave

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Received 22 November 1997; in revised form 29 June 1998

Abstract

In this study, the transient response of a finite crack subjected to an incident horizontally polarized shear wave and then propagated with a constant speed in an unbounded elastic solid is investigated. Initially, the finite crack with crack length l is stress-free and at rest. At time $t = 0$, an incident horizontally polarized shear wave strikes at one of the crack tips and will arrive at the other tip at a later time. Then, two crack tips propagate along the crack tip line with different velocities as the corresponding stress intensity factors reach their fracture toughness. The correspondent configuration is shown in Fig. 1. In analyzing this problem, diffracted waves generated by two propagating crack tips must be taken into account and it makes the analysis extremely difficult. In order to solve this problem, the transform formula in the Laplace transform domain between moving and stationary coordinates is first established. Complete solutions are determined by superposition of proposed fundamental solutions in the Laplace transform domain. The fundamental solutions to be used are from the problems of applying exponentially distributed traction and screw dislocation on crack faces and along the crack tip line, respectively. The exact transient solutions of dynamic stress intensity factor for the first few diffracted waves that arrive at two crack tips are obtained and expressed in compact formulations. Numerical calculations of dynamic stress intensity factors for both tips are evaluated and the results are discussed in detail. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Recently, the transient response of a solid medium containing a crack-like flaw under dynamic loads has received much attention. Scattering of elastic waves by cracks has attracted attention over the years for its importance towards the nondestructive evaluation of cracked bodies and the dynamic fracture analysis of materials. The interaction of a stress wave with a crack is a complicated problem and the analysis is mainly restricted to relatively simple problems. Most of the work, however, has been directed towards the solution of problems with a semi-infinite crack subjected

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PII: S0020-7683(98)00207-8

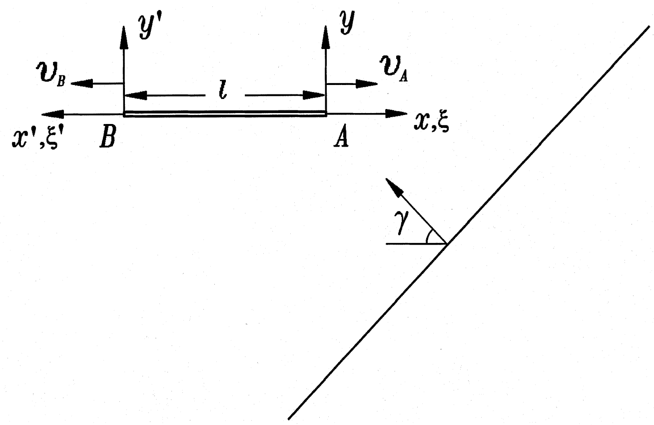

Fig. 1. Configuration and coordinate systems of a finite crack in an unbounded medium.

to distributed impact loading on crack faces. The complete solutions mentioned above can be obtained by integral transform methods in conjunction with direct application of the Wiener–Hopf technique (Noble, 1958) and the Cagniard–de Hoop method (de Hoop, 1958) of Laplace inversion. If the cracked problem having a characteristic length or loading condition is unsymmetrical, then the usual procedure using integral transform methods does not apply.

The stress intensity factors of a stationary finite crack upon diffraction of a time-harmonic wave have been obtained by Loeber and Sih (1968) and Sih and Loeber (1968, 1969). If integral transforms are applied to solve the transient response of a finite crack subjected to dynamic loading, a relationship among sectionally analytic function will be obtained which is more complicated than the form of the standard Wiener–Hopf equations. The generalized Wiener–Hopf equation can be solved iteratively to obtain the complete transient solution, and only the first step in the iteration process has been carried out. Thau and Lu (1971), following the work of Kostrov (1964) and Flitman (1963), treated the analogous transient problem of diffraction of an arbitrary plane dilatational wave by a stationary finite crack and a stationary finite rigid ribbon in an infinite elastic solid from the iteration process. Their results are exact only at the time interval that the dilatational wave has traveled the length of the crack twice. Sih and Embley (1972) have studied the near tip solution of a stationary finite crack under transient in-plane loading. They reduced the mixed boundary value problem to a standard Fredholm integral equation and subsequently inverted the Laplace transform of the stress components by a combination of numerical means and an application of the Cagniard inversion technique. A class of problems involving interaction between a stationary finite crack and other boundaries was considered by Chen (1977, 1978) and Itou (1980, 1981). With the exception of Loeber and Sih who considered the time-harmonic incident wave, all of the authors mentioned have simplified their problems by assuming symmetrically distributed loading conditions, and finally used a numerical Laplace inversion technique to obtain the solutions in the physical domain. Because of the mathematical difficulties, the closed form analytical solution for the problem of a finite crack subjected to transient waves is very rare.

The problem of an unbounded medium containing a stationary semi-infinite crack subjected to a pair of concentrated inplane loadings on the crack faces has been investigated by Freund (1974).

A straightforward application of the Wiener–Hopf method is not successful and the transient solution of stress intensity factor was obtained by Freund (1974) by an indirect approach based on the superposition of moving dislocations. He proposed a fundamental solution arising from an edge dislocation climbing along the line ahead of the crack tip with a constant speed to overcome the difficulties of the case with a characteristic length. The solution can be constructed by taking an integration over a climbing dislocation of different moving velocity. Basing his procedure on this method, Brock (1982, 1984), Brock et al. (1985), and Ma and Hou (1990, 1991) have analyzed a series of problems of a semi-infinite crack subjected to impact loading on crack faces. A thorough summary of the application of main direct methods of analysis for transient problems in dynamic fracture for elastic or inelastic problems has been given by Freund (1990). Freund (1990) has suggested an alternate approach based on the aforementioned moving dislocation solution to examine the same finite-crack problem that had been solved by Thau and Lu (1971). In practice, however, the alternate approach provided a solution that is valid for the same time range as before.

Kostrov (1966) and Achenback (1970a, b) have used the method based on Green's function to solve the problems of crack propagation for anti-plane deformation. In their studies, the region of integration for the integral equation is in a complicated shape, generally being bounded by a hyperbola and a number of straight lines. For points ahead of the crack tip,the region of integration reduces to a triangular region and the stress in the plane of the crack can thus be determined without difficulty. However, for material points not on the crack tip line, the region of integration is very complicated and careful analysis is needed. Scattering of plane harmonic waves by a running crack of finite length was investigated by Chen and Sih (1975). They found the dynamic stress intensity factors and crack opening displacements of the finite crack. Exact transient closed form solutions for a stationary semi-infinite crack subjected to a suddenly applied dynamic body force in an unbounded medium have been obtained by Tsai and Ma (1992) for the in-plane case and by Ma and Chen (1993) for the anti-plane case. They determined the transient full field solutions by superimposing a fundamental solution in the Laplace transform domain. The fundamental solution used in the problem is an exponentially distributed traction in the Laplace transform domain on the crack faces. This fundamental solution has also successfully been applied to solve the problems of a half plane containing a semi-infinite inclined crack by Tsai and Ma (1993) and Ma and Chen (1994) for in-plane and anti-plane problems respectively. Brock (1975) has studied the transient response for diffraction of an incident horizontally polarized shear wave by a stationary finite crack. His results indicated that the peak dynamic stress intensity factors could occur after the arrival of the second wave, which means that secondary diffractions may produce even higher peaks than the earlier peaking. Ing and Ma (1997) also investigated the same problem solved by Brock (1975) for the long time behavior by using superposition of new fundamental solutions in the Laplace transform domain. Their results, however, indicate that the maximum dynamic stress intensity factor in the transient period always occurs at the instance that the second wave arrives at the right or the left crack tip.

In this study, the transient response of a finite crack subjected to an incident plane horizontally polarized shear wave and then propagates after some delay time, is investigated. The geometrical configuration is shown in Fig. 1. If the stress intensity factor of the stationary crack tip is greater than the fracture toughness of the material, then it is assumed that the crack tip will start to propagate along the crack tip line with a constant velocity. Both tips, however, can propagate with different velocities. The propagation of crack with finite length can simulate dynamic fracture problem more realistically. In analyzing this problem, the waves diffracted between two stationary (and propagating) crack tips will make the analysis extremely difficult. It is impossible to solve this complicated problem by direct application of the standard Wiener–Hopf technique and some other approach must be followed. Two useful fundamental problems are proposed and used to overcome these difficulties. The proposed fundamental problems, which form a key element in the analysis, are solved exactly by the Wiener–Hopf method. The proposed fundamental problems are the problems of applying exponentially distributed traction and screw dislocation on crack faces and along the crack tip line, respectively. The first few waves diffracted by the stationary and propagating crack tips are constructed by superposition of the proposed fundamental solutions. Since the stress intensity factor is the key parameter in characterizing dynamic crack growth, we will focus our attention mainly on the determination of the dynamic stress intensity factor.

2. Proposed fundamental problems and fundamental solutions

Two alternative fundamental problems will be proposed and solved in this section which can then be used to construct the solution for the problem of a finite crack subjected to plane polarized shear waves. The solutions of an exponentially distributed traction applied at the propagating crack faces and exponentially distributed screw dislocations generated along the crack tip line in the Laplace transform domain will be referred to as the fundamental solutions. The diffracted waves scattered from the crack tips can be constructed by superimposing the proposed fundamental solutions in the Laplace transform domain.

Consider a fundamental problem of anti-plane deformation for a semi-infinite crack propagating in an unbound medium. The crack propagates along the crack line with a constant velocity v which is less than the shear wave speed of the material. In analyzing this problem, it is convenient to express the governing equation of wave motion in the moving coordinates $\xi - y$ as follows

$$(1-b^2v^2)\frac{\partial^2 w}{\partial \xi^2}+\frac{\partial^2 w}{\partial y^2}+2b^2v\frac{\partial^2 w}{\partial \xi \partial t}-b^2\frac{\partial^2 w}{\partial t^2}=0, \tag{1}$$

where w is the out-of plane displacement, and b is the slowness of the shear wave given by

$$b=\frac{1}{c_s}=\sqrt{\frac{\rho}{\mu}}.$$

Here c_s is the shear wave speed, μ and ρ are the respective shear modulus and the mass density of the material. The coordinate ξ defined by $\xi = x - vt$ is fixed with respect to the moving crack tip. The nonvanishing shear stresses are

$$\tau_{yz}=\mu\frac{\partial w}{\partial y},\quad \tau_{xz}=\mu\frac{\partial w}{\partial x}. \tag{2}$$

2.1. Fundamental solution of distributed loads on crack faces

We consider first that exponentially distributed tractions in the Laplace transformation domain are applied on the upper and lower crack faces of a propagating semi-infinite crack. Because the tractions are equal but opposite on the two crack faces, the problem can be viewed as a half-plane problem with the material occupying the region $y \geqslant 0$, and subjected to the following mixed boundary conditions in the Laplace transform domain

$$\bar{\tau}_{yz}(\xi, 0, s) = e^{s\eta\xi} \quad \text{for} \quad -\infty < \xi < 0, \tag{3}$$

$$\bar{w}(\xi, 0, s) = 0 \quad \text{for} \quad 0 < \xi < \infty. \tag{4}$$

The Laplace transform parameter s is taken as a positive number and η is a constant. The overbar symbol is used for denoting the transform on time t. This fundamental problem can be solved by the application of the standard integral transform method. Applying the one-sided Laplace transform over time, the two-sided Laplace transform over ξ under the restriction of $Re(\eta) > Re(\lambda)$, finally the Wiener–Hopf technique is implemented. The solutions of stresses and displacement in the Laplace transform domain, for the boundary conditions (3) and (4), can be expressed as follows

$$\bar{\tau}_{yz}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{\alpha_+^*(\lambda) e^{-s(\alpha^* y - \lambda\xi)}}{\alpha_+^*(\eta)(\eta - \lambda)} d\lambda, \tag{5}$$

$$\bar{\tau}_{xz}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{-\lambda e^{-s(\alpha^* y - \lambda\xi)}}{\alpha_-^*(\lambda)(\eta - \lambda)\alpha_+^*(\eta)} d\lambda, \tag{6}$$

$$\bar{w}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{-e^{-s(\alpha^* y - \lambda\xi)}}{\mu s \alpha_-^*(\lambda)(\eta - \lambda)\alpha_+^*(\eta)} d\lambda, \tag{7}$$

where

$$\alpha^*(\lambda) = \alpha_+^*(\lambda)\alpha_-^*(\lambda) = \sqrt{b + \lambda(1 - bv)}\sqrt{b + \lambda(1 + bv)}. \tag{8}$$

To ensure $Re(\alpha^*) \geqslant 0$ everywhere in the λ-plane, branch cuts are introduced from $b/(1 + bv)$ to ∞, and $-b/(1 - bv)$ to $-\infty$. The corresponding result of the dynamic stress intensity factor expressed in the Laplace transform domain is

$$\bar{K}(s) = \lim_{\xi \to 0} \bar{\tau}_{yz}(\xi, 0, s) = \frac{-\sqrt{2(1 - bv)}}{\sqrt{s}\alpha_+^*(\eta)}. \tag{9}$$

2.2. Fundamental solution of screw dislocation distributed along the crack tip line

Consider a semi-infinite crack contained in an unbounded medium. A distributed screw dislocation ahead of the crack tip line yields the following boundary conditions in the Laplace transform domain

$$\bar{w}(\xi, 0, s) = e^{s\eta\xi} \quad \text{for} \quad 0 < \xi < \infty, \tag{10}$$

$$\bar{\tau}_{yz}(\xi, 0, s) = 0 \quad \text{for} \quad -\infty < \xi < 0. \tag{11}$$

The particular problem posed can be solved by means of the Wiener–Hopf method. The solutions of stresses and the displacement expressed in the Laplace transform domain are

$$\bar{\tau}_{yz}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{\mu s \alpha_{-}^{*}(\eta)\alpha_{+}^{*}(\lambda)\mathrm{e}^{-s(\alpha^{*}y-\lambda\xi)}}{\eta-\lambda}\mathrm{d}\lambda, \tag{12}$$

$$\bar{\tau}_{xz}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{-\mu s \lambda \alpha_{-}^{*}(\eta)\mathrm{e}^{-s(\alpha^{*}y-\lambda\xi)}}{\alpha_{-}^{*}(\lambda)(\eta-\lambda)}\mathrm{d}\lambda, \tag{13}$$

$$\bar{w}(\xi, y, s) = \frac{1}{2\pi i}\int \frac{-\alpha_{-}^{*}(\eta)\mathrm{e}^{-s(\alpha^{*}y-\lambda\xi)}}{\alpha_{-}^{*}(\lambda)(\eta-\lambda)}\mathrm{d}\lambda, \tag{14}$$

The corresponding result of stress intensity factor expressed in the Laplace transform domain is

$$\bar{K}(s) = -\mu\sqrt{2s(1-bv)}\alpha_{-}^{*}(\eta). \tag{15}$$

3. Coordinate transformation relations in the Laplace transform domain

The superposition method can be applied successfully only if the fundamental solutions and the integral function of superposition are described in the same coordinate system. If they are not defined in the same coordinate system, they have to transform into the same one. Consider two moving coordinate systems (ξ, y) and (ξ', y) whose extending velocities are v_{A} and v_{B}, respectively, i.e. $\xi = x - v_{\mathrm{A}}t$ and $\xi' = x - v_{\mathrm{B}}t$. If a function described in the (ξ, y) coordinate system is represented in the Laplace transform domain as

$$\bar{Q}(\xi, y, s) = s^{n}\mathrm{e}^{sa}\int F(\lambda)\mathrm{e}^{-s\alpha_{\mathrm{A}}^{*}(\lambda)y+s\lambda\xi}\mathrm{d}\lambda, \tag{16}$$

where n is an arbitrary integer, and

$$\alpha_{A}^{*}(\lambda) = \alpha_{A+}^{*}(\lambda)\alpha_{A-}^{*}(\lambda) = \sqrt{b+\lambda(1-bv_{A})}\sqrt{b-\lambda(1+bv_{A})}.$$

Then it can be transformed into the (ξ', y) coordinate system with the following form

$$\bar{Q}(\xi', y, s) = -s^{n}\,\mathrm{e}^{sa}\int[1-\lambda(v_{\mathrm{A}}-v_{\mathrm{B}})]^{n-1}F\left(\frac{-\lambda}{1-\lambda(v_{\mathrm{A}}-v_{\mathrm{B}})}\right)\mathrm{e}^{-s\alpha_{\mathrm{B}}^{*}(\lambda)y+s\lambda[\xi-a(v_{\mathrm{A}}-v_{\mathrm{B}})]}\,\mathrm{d}\lambda, \tag{17}$$

in which

$$\alpha_{\mathrm{B}}^{*}(\lambda) = \alpha_{\mathrm{B}+}^{*}(\lambda)\alpha_{\mathrm{B}-}^{*}(\lambda) = \sqrt{b+\lambda(1-bv_{\mathrm{B}})}\sqrt{b-\lambda(1+bv_{\mathrm{B}})}.$$

The transformation relations described in eqns (16) and (17) can be proved if one inverses these two equations to time domain.

4. Dynamic stress intensity factors of two propagating crack tips

The evaluation of the stress intensity factor for a cracked body is a well-established concept in fracture mechanics, and it represents the cornerstone of linear elastic fracture mechanics. We will focus our attentions in this study mainly on the evaluation of the dynamic stress intensity factor.

A specific geometry to be considered here is an infinite medium containing a finite crack of length l as shown in Fig. 1. The origins of two stationary coordinate systems (x, y) and (x', y') are located at crack tips A and B, respectively. At time $t = 0$, an incident horizontally polarized shear wave arrives at the crack tip A, and then, two crack tips will propagate along the crack tip line with different velocities as the corresponding stress intensity factors reach its fracture toughness. The incident plane wave with an incident angle γ is represented by the general form

$$w^i(x, y, t) = F(t+bx\cos\gamma - by\sin\gamma), \tag{18}$$

where

$$F(t) = H(t)\int_0^t f(\tau)\,\mathrm{d}\tau, \tag{19}$$

in which F is identically zero when its argument is negative, but is otherwise an arbitrary wave form. Thus, the medium ahead of the incident plane wave front is undisturbed. In eqn (19), $H(\,)$ denotes the Heaviside step function and γ is the angle of the negative x-axis and the normal of the wavefront. The position of the wavefront for time $t < 0$ is also shown in Fig. 1. Here the angle γ is restricted to the range $0 < \gamma \leqslant \pi/2$.

At time $t = 0$, the incident plane wavefront strikes the crack tip A and will generate plane reflected and cylindrical diffracted waves. Some time later, i.e., $t = bl\cos\gamma$, the incident plane wave will arrive at the crack tip B and another diffracted wave will be induced. It is assumed that each crack tip will propagate along the crack tip line if the dynamic stress intensity factor of the tip reaches its fracture toughness K_c. The diffracted waves induced from one crack tip will propagate toward the other crack tip at a later time, and it makes the problem more difficult to solve because many waves will be generated from both tips. An effective superposition scheme will be proposed in this study to solve this complicated problem.

The incident horizontally polarized shear wave expressed in eqn (18) will give rise to the following shear stress in the infinite medium:

$$\tau^i_{yz}(x, y, t) = -\mu b\sin\gamma f(t+bx\cos\gamma - by\sin\gamma)H(t+bx\cos\gamma - by\sin\gamma). \tag{20}$$

Consider an incident step-stress wave for which

$$f(t) = \frac{\tau_0}{\mu b}. \tag{21}$$

Then the incident stress filed eqn (20) can be represented in the Laplace transform domain as

$$\bar{\tau}^i_{yz}(x, y, s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{\tau_0 \sin\gamma}{s(\lambda - b\cos\gamma)} \mathrm{e}^{-s\lambda y\tan\gamma + s\lambda x}\,\mathrm{d}\lambda, \tag{22}$$

or expressed in the (x', y') coordinate system as

$$\bar{\tau}^i_{y'z'}(x', y', s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{\tau_0 \sin\gamma}{s(\lambda - b\cos\gamma)} \mathrm{e}^{s\lambda y'\tan\gamma + s\lambda(x'+l)}\,\mathrm{d}\lambda. \tag{23}$$

Before the incident stress wave diffracted from the crack tip B, the stress field is precisely the same as that derived for a stationary semi-infinite crack lies in the plane $y = 0$ and $-\infty < x < 0$, and is struck by the same incident plane wave. The incident stress field $\bar{\tau}^i_{yz}(x, 0, s)$ at $y = 0$ generated by the step-stress shear wave is

$$\bar{\tau}^i_{yz}(x,0,s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{\tau_0 \sin\gamma}{s(\lambda - b\cos\gamma)} \mathrm{e}^{s\lambda x}\,\mathrm{d}\lambda. \tag{24}$$

The applied traction on the crack face, in order to eliminate the incident wave as indicated in eqn (24), has the functional form $\mathrm{e}^{s\lambda x}$. Since the solutions of applying traction $\mathrm{e}^{s\eta x}$ on stationary crack faces have been solved in Section 2 by setting $v = 0$, the reflected and diffracted fields can be constructed by superimposing the incident wave traction that is equal and opposite to eqn (24). When we combine eqns (7) and (24) (by setting $v = 0$), the solution of displacement $\bar{w}^{\mathrm{A}1d}$ for A1d wave (the first wave diffracted from the stationary crack tip A) in the upper plane can be expressed in the Laplace transform domain as follows

$$\begin{aligned}\bar{w}^{\mathrm{A}1d}(x,y,s) &= \frac{-1}{2\pi i}\int_{\Gamma_{\eta 1}} \frac{\tau_0 \sin\gamma}{s(\eta_1 - b\cos\gamma)} \frac{1}{2\pi i}\int_{\Gamma_{\eta 2}} \frac{-\mathrm{e}^{-s\alpha y + s\eta_2 x}}{\mu s\alpha_+(\eta_1)(\eta_1-\eta_2)\alpha\_(\eta_2)}\mathrm{d}\eta_2\mathrm{d}\eta_1 \\ &= \frac{\sqrt{2}\tau_0\sin(\gamma/2)}{\mu\sqrt{b}s^2}\frac{1}{2\pi i}\int_{\Gamma_\lambda}\frac{\mathrm{e}^{-s\alpha y + s\lambda x}}{\alpha\_(\lambda)(\lambda - b\cos\gamma)}\mathrm{d}\lambda. \end{aligned} \tag{25}$$

The corresponding stress intensity factor expressed in the Laplace transform domain is

$$\begin{aligned}\bar{K}^{\mathrm{A}1d}(s) &= \frac{-1}{2\pi i}\int_{\Gamma_\lambda}\frac{\tau_0\sin\gamma}{s(\lambda - b\cos\gamma)}\left\{\frac{-\sqrt{2}}{\sqrt{s}\alpha_+(\lambda)}\right\}\mathrm{d}\lambda \\ &= \frac{-2\tau_0\sin(\gamma/2)}{s^{3/2}\sqrt{b}}. \end{aligned} \tag{26}$$

By using the Cagniard–de Hoop method of Laplace inversion, the dynamic stress intensity factor at the crack tip A induced by the incident wave expressed in time domain will be

$$K^{\mathrm{A}1d}(t) = -4\tau_0\sqrt{\frac{t}{\pi b}}\sin(\gamma/2)H(t) \tag{27}$$

Equation (27) is a well known solution of a stationary semi-infinite crack subjected to an incident plane wave. After some delay time t_f^{A}, the dynamic stress intensity factor of tip A may reach its fracture toughness K_c, and the tip will begin to propagate. The delay time t_f^{A} can be determined from eqn (27) as follows

$$\left|-4\tau_0\sqrt{\frac{t_f^{\mathrm{A}}}{\pi b}}\sin(\gamma/2)\right| = K_c,$$

so

$$t_f^{\mathrm{A}} = \pi b\left(\frac{K_c}{4\tau_0 \sin(\gamma/2)}\right)^2. \tag{28}$$

In this study, it is assumed that the incident plane shear wave will always cause the crack tip A to propagate along the crack tip line. Consequently, the fracture toughness K_c must be less than the maximum amplitude of dynamic stress intensity factor of tip A, i.e., $K_c \leqslant K_{\max}^{\mathrm{A}}(t)$. However, it was also known from Ing and Ma (1997) that the stress intensity factor of tip A will arrive at its maximum amplitude at time $t = bl(1+\cos\gamma)$. So, we have the condition that

$$K_c \leqslant |K_{\max}^{\mathrm{A}1d}(t)| = 2\tau_0\sqrt{\frac{2l}{\pi}}\sin\gamma. \tag{29}$$

At time $t = t_f^{\mathrm{A}}$, the dynamic stress intensity factor of crack tip A reaches its critical value and this tip starts to propagate with a constant velocity v_{A}. The incident wave written in the Laplace transform domain for the moving coordinate system (ξ, y) will have the following form

$$\bar{\tau}_{yz}^{i}(\xi, y, s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{\tau_0 \sin\gamma(1+bv_{\mathrm{A}}\cos\gamma)}{s[(1+bv_{\mathrm{A}}\cos\gamma)\lambda - b\cos\gamma]} \mathrm{e}^{-s\lambda y\tan\gamma + s\lambda(\xi - v_{\mathrm{A}}t_f^{\mathrm{A}})}\,\mathrm{d}\lambda, \tag{30}$$

where $\xi = x - v_{\mathrm{A}}(t - t_f^{\mathrm{A}})$. The applied traction on crack faces as expressed in eqn (30), has the functional form $\mathrm{e}^{s\lambda\xi}$. The diffracted field generated from the propagating crack tip A can be constructed by superimposing the fundamental solution and the stress distribution in eqn (30). The result of displacement expressed in the Laplace transform domain will be

$$\begin{aligned}\bar{w}^{\mathrm{A}1v}(\xi, y, s) &= \frac{-1}{2\pi i}\int_{\Gamma_{\eta 1}} \frac{\tau_0 \sin\gamma(1+bv_{\mathrm{A}}\cos\gamma)\,\mathrm{e}^{-s\eta_1 v_{\mathrm{A}} t_f^{\mathrm{A}}}}{s[(1+bv_{\mathrm{A}}\cos\gamma)\eta_1 - b\cos\gamma]} \frac{1}{2\pi i}\int_{\Gamma_{\eta 2}} \frac{-\mathrm{e}^{-s\alpha_{\mathrm{A}}^{*}y + s\eta_2\xi}}{\mu s\alpha_{\mathrm{A}+}^{*}(\eta_1)(\eta_1 - \eta_2)\alpha_{\mathrm{A}-}^{*}(\eta_2)}\,\mathrm{d}\eta_2\,\mathrm{d}\eta_1 \\ &= \frac{\sqrt{2}\tau_0 \sin(\gamma/2)(1+bv_{\mathrm{A}}\cos\gamma)^{3/2}}{\mu\sqrt{b}s^2} \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{\mathrm{e}^{-s\alpha_{\mathrm{A}}^{*}y + s\lambda\xi}}{\alpha_{\mathrm{A}-}^{*}(\lambda)[1+bv_{\mathrm{A}}\cos\gamma)\lambda - b\cos\gamma]}\,\mathrm{d}\lambda.\end{aligned} \tag{31}$$

The dynamic stress intensity factor for a propagating crack in an infinite medium can also be constructed by a similar manner. The result in the Laplace transform domain can be obtained from eqns (10) and (30) and is expressed as follows

$$\begin{aligned}\bar{K}^{\mathrm{A}1v}(s) &= \frac{-1}{2\pi i}\int_{\Gamma_{\eta 1}} \frac{\tau_0 \sin\gamma(1+bv_{\mathrm{A}}\cos\gamma)\mathrm{e}^{-s\lambda v_{\mathrm{A}} t_f^{\mathrm{A}}}}{s[(1+bv_{\mathrm{A}}\cos\gamma)\lambda - b\cos\gamma]}\left\{\frac{-\sqrt{2(1-bv_{\mathrm{A}})}}{\sqrt{s}\alpha_{\mathrm{A}+}^{*}(\lambda)}\right\}\mathrm{d}\lambda \\ &= -\frac{\sqrt{2}\tau_0 \sin(\gamma/2)\sqrt{1+bv_{\mathrm{A}}\cos\gamma)(1-bv_{\mathrm{A}})}}{\sqrt{b}s^{3/2}}\mathrm{e}^{-sbv_{\mathrm{A}}t_f^{\mathrm{A}}\cos\gamma/1+bv_{\mathrm{A}}\cos\gamma}.\end{aligned} \tag{32}$$

The inversion Laplace transform of eqn (32) will have the following form

$$K^{\mathrm{A}1v}(t) = \frac{-4\tau_0 \sin(\gamma/2)\sqrt{(1+bv_{\mathrm{A}}\cos\gamma)(1-bv_{\mathrm{A}})}}{\sqrt{\pi b}}\sqrt{t - \frac{bv_{\mathrm{A}}t_f^{\mathrm{A}}\cos\gamma}{1+bv_{\mathrm{A}}\cos\gamma}}H(t - t_f^{\mathrm{A}}). \tag{33}$$

The Heaviside step function in eqn (33) results from the effect that $K^{A1v}(t)$ is valid only for time $t > t_f^A$. The dynamic stress intensity factor expressed in eqn (33) is a well-known solution for a propagating semi-infinite crack subjected to an incident step-stress wave. The analogous solution has also been found by Ma and Burgers (1986) using a different method.

Subsequently, the incident plane wave will propagate toward the crack tip B and will be diffracted at time $t = bl\cos\gamma$. Following the similar procedure that is used for constructing the A1d wave, the B1d wave (the first wave diffracted from the stationary crack tip B) can be constructed in the coordinate system (x', y') by using eqns (23) and (7) (by setting $v = 0$) as follows

$$\bar{w}^{B1d}(x', y', s) = \frac{-1}{2\pi i}\int_{\Gamma_{\eta 1}} \frac{\tau_0 \sin\gamma e^{s\eta_1 l}}{s(\eta_1 + b\cos\gamma)} \frac{1}{2\pi i}\int_{\Gamma_{\eta 2}} \frac{-e^{-s\alpha y' + s\eta_2 x'}}{\mu s\alpha_+(\eta_1)(\eta_1 - \eta_2)\alpha\_(\eta_2)} d\eta_2 d\eta_1$$
$$= \frac{\sqrt{2}\tau_0 \cos(\gamma/2)\, e^{-sbl\cos\gamma}}{\mu\sqrt{b}s^2} \frac{-1}{2\pi i}\int_{\Gamma_\lambda} \frac{e^{-s\alpha y' + s\lambda x'}}{\alpha\_(\lambda)(\lambda + b\cos\gamma)} d\lambda. \tag{34}$$

The corresponding stress intensity factor at the crack tip B induced by the incident plane wave is

$$\bar{K}^{B1d}(s) = \frac{2\sqrt{b}\tau_0 \cos(\gamma/2) e^{-sbl\cos\gamma}}{s^{3/2}\sqrt{b}}. \tag{35}$$

The dynamic stress intensity factor at the crack tip B expressed in time domain will be

$$K^{B1d}(t) = 4\tau_0\sqrt{\frac{t - bl\cos\gamma}{\pi b}} \cos(\gamma/2) H(t - bl\cos\gamma). \tag{36}$$

The results expressed in eqns (27) and (36) are well-known solutions of dynamic stress intensity factor for the first two diffractions of a step-stress wave by a stationary finite crack in an unbounded medium. The same solutions have also been obtained by Achenbach (1970a) and Ing and Ma (1997) using different methods.

Similarly, after some delay time t_f^B, the crack tip B begins to propagate with a constant velocity v_B as the dynamic stress intensity factor exceeds its fracture toughness K_c. It is assumed in this study that the crack tip B starts to propagate before the A1d wave arrived the tip, i.e., $bl > t_f^B > bl\cos\gamma$. The delay time t_f^B can be obtained from eqn (36) as follows

$$4\tau_0\sqrt{\frac{t_f^B - \mathrm{bl}\cos\gamma}{\pi b}} \cos(\gamma/2) = K_c$$

and we have

$$t_f^B = \pi b\left(\frac{K_c}{4\tau_0\cos(\gamma/2)}\right)^2 + bl\cos\gamma. \tag{37}$$

Notice that the fracture toughness K_c must be less than $K_{max}^{B1d}(t)$, so we have

$$K_c \leqslant |K_{max}^{B1d}(t)| = 2\tau_0\sqrt{\frac{2l}{\pi}}\sin\gamma. \tag{38}$$

The result in eqn (38) is the same as that in eqn (29). Consequently, from eqns (29) and (38), we can obtain the maximum fracture toughness that allows both crack tips to propagate as follows

$$K_{c,\max} = 2\tau_0\sqrt{\frac{2l}{\pi}}\sin\gamma. \tag{39}$$

Following the same procedure, we represent the incident field in the moving coordinate system (ξ', y′) as

$$\bar{\tau}^{i}_{y'z'}(\xi',y',s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda}\frac{\tau_0\sin\gamma(1-bv_{\mathrm{B}}\cos\gamma)}{s[(1-bv_{\mathrm{B}}\cos\gamma)\lambda+b\cos\gamma]}\mathrm{e}^{s\lambda y'\tan\gamma+s\lambda(\xi'-v_{\mathrm{B}}t_f^{\mathrm{B}}+l)}\mathrm{d}\lambda, \tag{40}$$

where $\xi' = x'-v_{\mathrm{B}}(t-t_f^{\mathrm{B}})$. The displacement field and the stress intensity factor $K^{\mathrm{B}1v}(t)$ after the tip B starts to propagate can be obtained from eqns (40), (7) and (9), and the final results are

$$\bar{w}^{\mathrm{B}1v}(\xi',y',s) = \frac{\sqrt{2}\tau_0\cos(\gamma/2)(1-bv_{\mathrm{B}}\cos\gamma)^{3/2}}{\mu\sqrt{b}s^2}\mathrm{e}^{sb\cos\gamma(v_Bt_f^{\mathrm{B}}-l)/1-bv_B\cos\gamma}$$
$$\times\frac{-1}{2\pi i}\int_{\Gamma_\lambda}\frac{\mathrm{e}^{-s\alpha^*_{\mathrm{B}}y'+s\lambda\xi'}}{\alpha^*_{\mathrm{B}-}(\lambda)\,[(1-bv_{\mathrm{B}}\cos\gamma)\lambda+b\cos\gamma]}\mathrm{d}\lambda. \tag{41}$$

$$K^{\mathrm{B}1v}(t) = \frac{4\tau_0\cos(\gamma/2)\sqrt{(1-bv_{\mathrm{B}}\cos\gamma((1-bv_{\mathrm{B}})}}{\sqrt{\pi b}}\sqrt{t-\frac{b\cos\gamma(l-v_{\mathrm{B}}t_f^{\mathrm{B}})}{1-bv_{\mathrm{B}}\cos\gamma}}H(t-t_f^{\mathrm{B}}). \tag{42}$$

When the diffracted B1d or the B1v wave arrives at the right tip of the finite crack at a later time, it carries a discontinuous displacement in the z-direction which violates the boundary condition for $\xi > 0$. In order to satisfy the boundary condition where the displacement must be continuous for $\xi > 0$, a distributed screw dislocation is required to close the opening displacement ahead of the propagating crack tip. The diffracted A2d and A2v waves will be induced when the B1d and B1v waves arrive at the moving crack tip A, respectively. We change the formulation in eqns (34) and (41) to (ξ, y) coordinate system by using the transformation relations established in Section 3, then the displacements we must eliminate ahead of the propagating tip A are

$$\bar{w}^{\mathrm{B}1d}(\xi,0,s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda}\frac{\sqrt{2}\tau_0\cos(\gamma/2)\mathrm{e}^{-sbl\cos\gamma}\mathrm{e}^{s\lambda(\xi-v_{\mathrm{A}}t_f^{\mathrm{A}}+l+v_{\mathrm{A}}bl\cos\gamma)}}{\mu\sqrt{b}s^2(1-\lambda v_{\mathrm{r}})^3(1-\lambda_1v_{\mathrm{B}})^{3/2}\alpha^*_{\mathrm{B}-}(\lambda_1)\,[(1-bv_{\mathrm{B}}\cos\gamma)\lambda_1+b\cos\gamma]}\mathrm{d}\lambda, \tag{43}$$

$$\bar{w}^{\mathrm{B}1v}(\xi,0,s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda}\frac{\sqrt{2}\tau_0\cos(\gamma/2)(1-bv_{\mathrm{B}}\cos\gamma)^{3/2}\mathrm{e}^{-st_{\mathrm{B}}}\mathrm{e}^{s\lambda(\xi-v_{\mathrm{A}}t_f^{\mathrm{A}}-v_{\mathrm{B}}t_f^{\mathrm{B}}+l+v_{\mathrm{r}}t_{\mathrm{B}})}}{\mu\sqrt{b}s^2(1-\lambda v_{\mathrm{r}})^3\alpha^*_{\mathrm{B}-}(\lambda_1)\,[(1-bv_{\mathrm{B}}\cos\gamma)\lambda_1+b\cos\gamma]}\mathrm{d}\lambda, \tag{44}$$

where $v_{\mathrm{r}} = v_{\mathrm{A}}+v_{\mathrm{B}}$ is the relative velocity between two moving coordinate systems and

$$\lambda_1 = \frac{\lambda}{1-\lambda v_{\mathrm{r}}},\quad t_{\mathrm{B}} = \frac{b\cos\gamma(l-v_{\mathrm{B}}t_f^{\mathrm{B}})}{1-bv_{\mathrm{B}}\cos\gamma}.$$

Again we treat the crack as a propagating semi-infinite crack which lies along the line $y = 0$,

$-\infty < \xi < 0$. The diffracted A2d and A2v waves generated from the propagating crack tip A can be obtained by superimposing the distributed dislocation that equal and opposite to eqns (43) and (44) ahead of the tip $\xi > 0$ in the Laplace transform domain, respectively. Here we only analyze the corresponding stress intensity factors by using eqns (43), (44), and (15) as follows

$$\bar{K}^{\mathrm{A}2d}(s) = \frac{-1}{2\pi i}\int_{\Gamma_\lambda} \frac{\sqrt{2}\tau_0\cos(\gamma/2)\mathrm{e}^{-sbl\cos\gamma}\,\mathrm{e}^{s\lambda(l-v_\mathrm{A}t_f^\mathrm{A}+v_\mathrm{A}bl\cos\gamma)}}{\mu\sqrt{b}s^2(1-\lambda v_\mathrm{r})^3(1-\lambda_1 v_\mathrm{B})^{3/2}\alpha^*_{\mathrm{B}-}(\lambda_1)\,[(1-bv_\mathrm{B}\cos)\gamma\lambda_1+b\cos\gamma]}$$
$$\{-\mu\sqrt{2s(1-bv_\mathrm{A})}\alpha^*_{\mathrm{A}-}(\lambda)\}\,\mathrm{d}\lambda$$
$$= \frac{-1}{2\pi i}\int_{\Gamma_\lambda} \frac{2\tau_0\cos(\gamma/2)\sqrt{1-bv_\mathrm{A}}\,\mathrm{e}^{-sbl\cos\gamma}\alpha^*_{\mathrm{A}-}(\lambda)\,\mathrm{e}^{s\lambda(l-v_\mathrm{A}t_f^\mathrm{A}+v_\mathrm{A}bl\cos\gamma)}}{\sqrt{b}s^{3/2}(1-\lambda v_\mathrm{A})^{3/2}\alpha^*_{\mathrm{A}+}(\lambda)[(1+bv_\mathrm{A}\cos\gamma)\lambda-b\cos\gamma]}\mathrm{d}\lambda, \tag{45}$$

$$\bar{K}^{\mathrm{A}2v}(s) = \frac{-1}{2\pi i}\int_{\Gamma_\lambda} \frac{2\tau_0\cos(\gamma/2)\sqrt{1-bv_\mathrm{A}}(1-bv_\mathrm{B}\cos\gamma)^{3/2}\,\mathrm{e}^{-st_\mathrm{B}}\alpha^*_{\mathrm{A}-}(\lambda)\,\mathrm{e}^{s\lambda(l-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B})}}{\sqrt{b}s^{3/2}(1-\lambda v_\mathrm{r})^{3/2}\alpha^*_{\mathrm{A}+}(\lambda_1)[(1+bv_\mathrm{A}\cos\gamma)\lambda-b\cos\gamma]} \tag{46}$$

Inversion of the Laplace transform of eqn (45) yields

$$K^{\mathrm{A}2d}(t) = \frac{4\tau_0\cos(\gamma/2)\sqrt{1-bv_\mathrm{A}}}{\pi^{3/2}\sqrt{b}}\int_{\frac{b(l-v_\mathrm{A}t_f^\mathrm{A}+v_\mathrm{A}bl\cos\gamma)}{1-bv_\mathrm{A}}}^{t-bl\cos\gamma}$$
$$\times\frac{\sqrt{t-\tau-bl\cos\gamma}\sqrt{\tau+b[l+v_\mathrm{A}(\tau+bl\cos\gamma-t_f^\mathrm{A})]}}{[l+v_\mathrm{A}(\tau+bl\cos\gamma-t_f^\mathrm{A})]^{3/2}}$$
$$\times\frac{[l+v_\mathrm{A}(bl\cos\gamma-t_f^\mathrm{A})]^{3/2}}{\{\tau+b\cos\gamma[l+v_\mathrm{A}(\tau+bl\cos\gamma-t_f^\mathrm{A})]\}\sqrt{\tau-b[l+v_\mathrm{A}(\tau+bl\cos\gamma-t_f^\mathrm{A})]}}$$
$$\times\,\mathrm{d}\tau H(t-t_{\mathrm{A}2d}), \tag{47}$$

where $t_{\mathrm{A}2d}$ is the arrival time of the B1d wave at the crack tip A and it can be expressed as

$$t_{\mathrm{A}2d} = \frac{b(l-v_\mathrm{A}t_f^\mathrm{A}+v_\mathrm{A}bl\cos\gamma)}{1-bv_\mathrm{A}}+bl\cos\gamma = \frac{bl\cos\gamma+b(l-v_\mathrm{A}t_f^\mathrm{A})}{1-bv_\mathrm{A}}. \tag{48}$$

The inverse Laplace transform of eqn (46) is

$$K^{\mathrm{A}2v}(t) = \frac{4\tau_0\cos(\gamma/2)\sqrt{1-bv_\mathrm{A}}(1-bv_\mathrm{B}\cos\gamma)^{3/2}}{\pi^{3/2}\sqrt{b}}\int_{\frac{b(l-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B}}{1-bv_\mathrm{A}}}^{t-t_\mathrm{B}}$$
$$\times\frac{\sqrt{t-\tau-t_\mathrm{B}}}{(l-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B}+v_\mathrm{r}\tau)^{3/2}}\times\frac{\sqrt{(1+bv_\mathrm{A})\tau+b(l-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B})}}{[\tau+b\cos\gamma(l+v_\mathrm{A}\tau-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B})]}$$
$$\times\frac{(l-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B})^{3/2}}{\sqrt{(1-bv_\mathrm{A})\tau-b(l-v_\mathrm{A}t_f^\mathrm{A}-v_\mathrm{B}t_f^\mathrm{B}+v_\mathrm{r}t_\mathrm{B})}}\mathrm{d}\tau H(t-t_{\mathrm{A}2v}), \tag{49}$$

where t_{A2v} is the arrival time of the B1v wave at the propagating crack tip A and it can be obtained as

$$t_{A2v} = \frac{b(l - v_A t_f^A) + t_f^B}{1 - b v_A}. \tag{50}$$

In addition, eqn (49) is valid only for $l - v_A t_f^A - v_b t_f^B > 0$. But this is adaptable for most propagating speed except for the high speed cases ($v_A/c_s \to 1$, $v_B/c_s \to 1$).

Similarly, the B2d and the B2v diffracted waves scattering from the crack tip B will be induced after the A1d and the A1v waves passed the tip. We change eqns (25) and (31) to (ξ', y') coordinate system by using the transformation relations, and the displacement fields along the crack tip line in the Laplace transform domain will be

$$\bar{w}^{A1d}(\xi', 0, s) = \frac{-1}{2\pi i}\int_{\Gamma_\lambda} \frac{\sqrt{2}\tau_0 \sin(\gamma/2) e^{s\lambda(\xi' - v_B t_f^B + l)}}{\mu\sqrt{b} s^2 (1-\lambda v_r)^3 (1-\lambda_1 v_A)^{3/2} \alpha^*_{A-}(\lambda_1)\, [(1+b v_A \cos\gamma)\lambda_1 - b\cos\gamma]} d\lambda, \tag{51}$$

$$\bar{w}^{A1v}(\xi', 0, s) = \frac{-1}{2\pi i}\int_{\Gamma_\lambda} \frac{\sqrt{2}\tau_0 \sin(\gamma/2)(1+b v_A \cos\gamma)^{3/2}\, e^{-s t_A}\, e^{s\lambda(\xi' - v_A t_f^A - v_B t_f^B + l + v_r t_A)}}{\mu\sqrt{b} s^2 (1-\lambda v_r)^3 \alpha^*_{A-}(\lambda_1)[(1+b v_A \cos\gamma)\lambda_1 - b\cos\gamma]} d\lambda, \tag{52}$$

where

$$t_A = \frac{b v_A t_f^A \cos\gamma}{1 + b v_A \cos\gamma}. \tag{53}$$

Using the fundamental solution in eqn (15), the stress intensity factors corresponding to the B2d and the B2v waves can be obtained as follows

$$\bar{K}^{B2d}(s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{2\tau_0 \sin(\gamma/2)\sqrt{1-b v_B}\,\alpha^*_{B-}(\lambda) e^{s\lambda(l - v_B t_f^B)}}{\sqrt{b} s^{3/2} (1-\lambda v_B)^{3/2} \alpha^*_{B+}(\lambda)\, [(1-b v_B \cos\gamma)\lambda + b\cos\gamma]} d\lambda, \tag{54}$$

$$\bar{K}^{B2v}(s) = \frac{1}{2\pi i}\int_{\Gamma_\lambda} \frac{2\tau_0 \sin(\gamma/2)\sqrt{1-b v_B}(1+b v_A \cos\gamma)^{3/2}\, e^{-s t_A} \alpha^*_{B-}(\lambda)\, e^{s\lambda(l - v_A t_f^A - v_B t_f^B + v_r t_A)}}{\sqrt{b} s^{3/2} (1-\lambda v_r)^{3/2} \alpha^*_{B+}(\lambda_1)[(1-b v_B \cos\gamma)\lambda + b\cos\gamma]} d\lambda. \tag{55}$$

Applying the inverse Laplace transform to eqns (54) and (55), the dynamic stress intensity factors in time domain are

$$K^{B2d}(t) = \frac{-4\tau_0 \sin(\gamma/2)\sqrt{1-b v_B}}{\pi^{3/2}\sqrt{b}} \int_{\frac{b(l - v_B t_f^B)}{1 - b v_B}}^{t} \frac{\sqrt{t-\tau}\sqrt{\tau + b[l + v_B(\tau - t_f^b)]}}{[l + v_B(\tau - t_f^B)]^{3/2}}$$
$$\times \frac{(l - v_B t_f^B)^{3/2}}{\{\tau - b\cos\gamma[l + v_B(\tau - t_f^B)]\}\sqrt{\tau - b[l + v_B(\tau - t_f^B)]}} d\tau H(t - t_{B2d}), \tag{56}$$

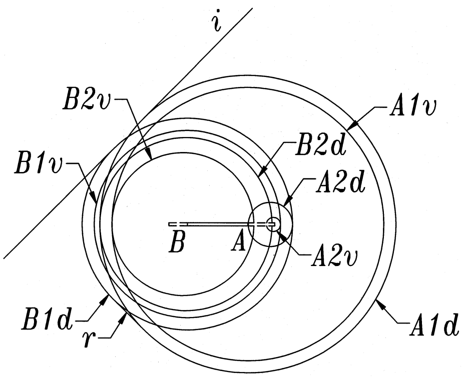

Fig. 2. Wave fronts of the incident and diffracted waves for a short time period.

$$K^{B2v}(t) = \frac{-4\tau_0 \sin(\gamma/2)\sqrt{1-bv_B}(1+bv_A\cos\gamma)^{3/2}}{\pi^{3/2}\sqrt{b}} \int_{\frac{b(l-v_A t_f^A - v_B t_f^B + v_r t_A)}{1-bv_B}}^{t-t_A} \frac{\sqrt{t-\tau-t_A}}{(l-v_A t_f^A - v_B t_f^B + v_r t_A + v_r \tau)^{3/2}} \times \frac{\sqrt{(1+bv_B)\tau + b(l-v_A t_f^A - v_B t_f^B + v_r t_A)}}{[\tau - b\cos\gamma(l+v_B\tau - v_A t_f^A - v_B t_f^B + v_r t_A)]} \times \frac{(l-v_A t_f^A - v_B t_f^B + v_r t_B)^{3/2}}{\sqrt{(1-bv_B)\tau - b(l-v_A t_f^A - v_B t_f^B + v_r t_A)}} \mathrm{d}\tau H(t-t_{B2v}), \tag{57}$$

where t_{B2d} and t_{B2v} are the arrival times of the A1d and the A1v waves, respectively, and

$$t_{B2d} = \frac{b(l-v_B t_f^B)}{1-bv_B}, \tag{58}$$

$$t_{B2v} = \frac{b(l-v_B t_f^B) + t_f^A}{1-bv_B}. \tag{59}$$

5. Numerical results

In the previous section, the transient solutions of dynamic stress intensity factors for the first few diffractions of a horizontally polarized shear wave by a propagating finite crack have been derived. The induced wave fronts of incident and diffracted waves in a short time period are shown in Fig. 2. Figures 3 and 4 show the dimensionless stress intensity factors K^A/K_c and K^B/K_c versus the dimensionless time t/bl for different values of the incident angle γ at crack tips A and B, respectively. It indicates in Fig. 3 that the dynamic stress intensity factors at crack tip A will increase as the incident angles increase. However, the stress intensity factors at crack tip B increase as the incident angles decrease after the first four waves passed the tip. That is, the dynamic

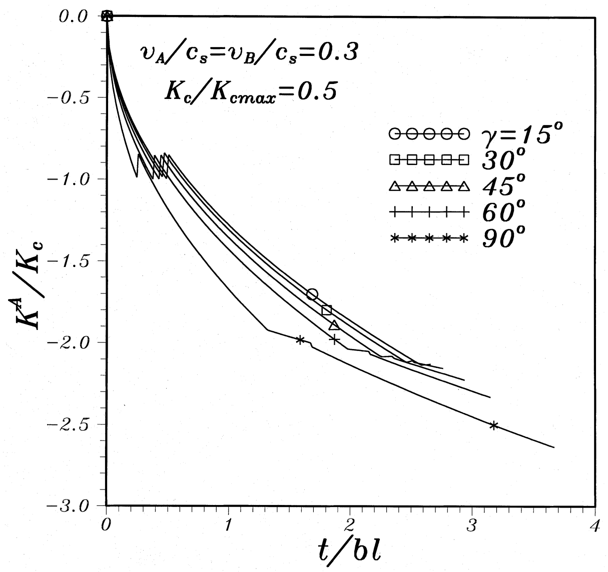

Fig. 3. The dynamic stress intensity factor K^A/K_c for different values of the incident angle γ.

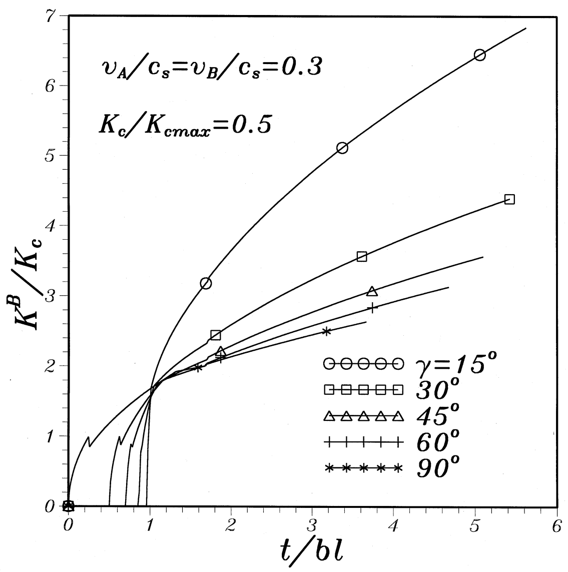

Fig. 4. The dynamic stress intensity factor K^B/K_c for different values of the incident angle γ.

stress intensity factor at crack tip B is much greater than that at crack tip A for the same incident angle.

Figure 5 shows the dimensionless stress intensity factors K^A/K_c and K^B/K_c versus the dimensionless time t/bl for different values of fracture toughness. It can be seen that the ratios for K^A/K_c

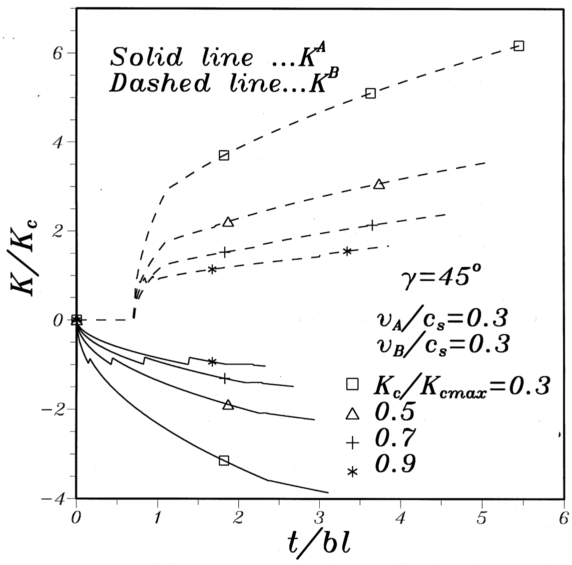

Fig. 5. The dynamic stress intensity factor for different values of fracture toughness.

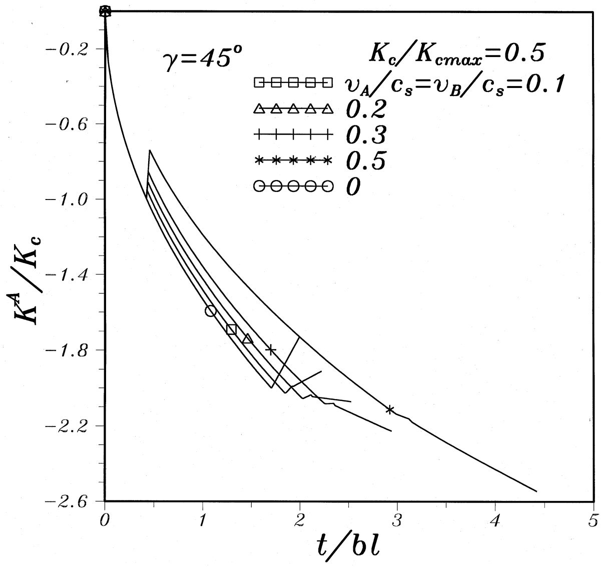

Fig. 6. The dynamic stress intensity factor K^A/K_c for various crack propagating velocities.

and K^B/K_c both increase rapidly for smaller K_c after the crack begins to propagate. It means that for larger K_c, the crack may stop propagating after it has propagated for a period of time. Moreover, Fig. 5 also indicates that the dynamic stress intensity factor at crack tip B is larger than that at crack tip A for the same value of fracture toughness. Figures 6 and 7 show the dimensionless

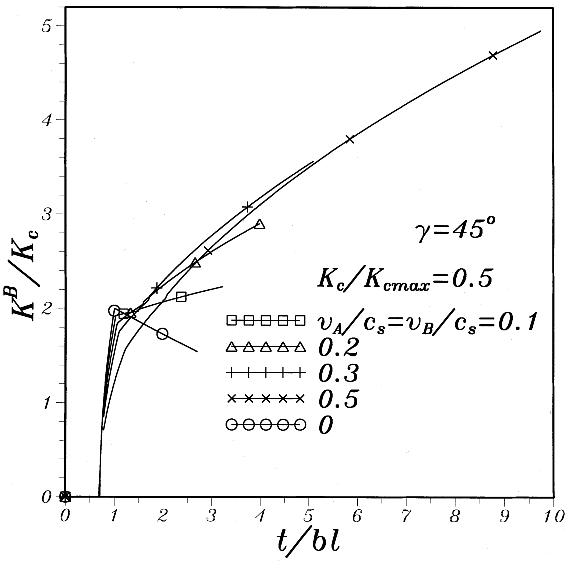

Fig. 7. The dynamic stress intensity factor K^B/K_c for various crack propagating velocities.

stress intensity factors K^A/K_c and K^B/K_c versus the dimensionless time t/bl for different crack propagating velocities, respectively. It shows that the influence of secondary diffraction on dynamic stress intensity factor for higher velocity is relatively smaller than that for lower velocity.

6. Conclusions

Most of the problems that have been studied in the development of fracture mechanics are quasi-static. Numerous problems have existed for which the assumption that the deformation is quasi-static is invalid and the inertia of the material must be taken into account. Because of the difficulties in mathematical complexity, analytical solutions for an elastic solid containing a finite crack subjected to dynamic loading are very rare. In conventional studies of a semi-infinite crack in an unbounded medium subjected to dynamic loading, the complete solution can be obtained by applying direct integral transform methods. If a cracked body having a characteristic length or the loading condition is unsymmetrical, then the same procedure can not be applied directly. In this investigation, we propose a powerful superposition methodology, which is performed in the Laplace transform domain, and successfully applied to solve the transient response of a finite crack propagating in an unbounded medium. The finite crack is stuck by a horizontally polarized shear wave. After some delay time, two stationary crack tips will start to propagate along the crack tip line with constant velocity as the stress intensity factor reaches its fracture toughness. Two useful fundamental solutions and the coordinate transformation relations are proposed to solve this problem. The first few waves diffracted by the stationary and propagating crack tips are obtained and expressed in very compact formulations.

It is interesting to note that the dynamic stress intensity factor during crack propagation of crack tip B is larger than that of crack tip A, which is the tip that the incident plane wave first strikes. It means that after crack starts to propagate, crack tip A is easier to arrest than crack tip B. Furthermore, it also indicates in this study that the influence of secondary diffracted waves on dynamic stress intensity factor for higher propagation velocity is relatively smaller than for lower velocity.

Acknowledgements

The authors gratefully acknowledge the financial support of this research by the National Science Council (Republic of China) under Grant NSC 83-0401-E-002-115.

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